Bayesian methods for ecological and environmental modelling

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What we will cover today:

Session 2a: Some more Bayesian basics

- Hypothesis testing
- Prediction

Session 2b MCMC

- Sampling concepts

Session 3 Linear modelling

- Linear regression as a Bayesian

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Session 2a
Some more Bayesian basics

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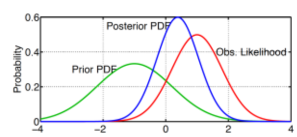
Recap

Bayes rule for probability densities

$$P(\theta|y) = \frac{P(y|\theta) \cdot P(\theta)}{P(y)} \propto P(y|\theta) \cdot P(\theta)$$



Note that if prior is uninformative, the posterior returns the likelihood

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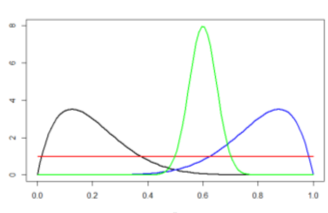
Recap

From the practical:

- Different probability distributions
- Using commands to calculate likelihoods and draw random values
- Importance of prior distributions



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Motivation

Bayesian inference is an approach to solving two general types of problem

1. Understanding and quantifying relationships
2. Prediction

To achieve the objectives, we:

1. Collect (or generate) data
2. Formulate a probability model

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Hypothesis Testing

Point estimate for parameter

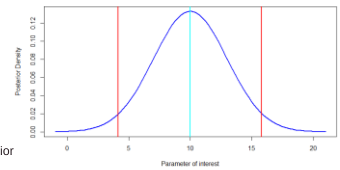
Mean, median or mode of posterior

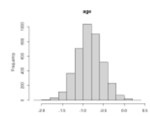
Interval estimate

Central posterior interval –based on posterior quantiles

How do I get my p-value???

Remember what we said yesterday - we can now talk in terms of probability of parameters





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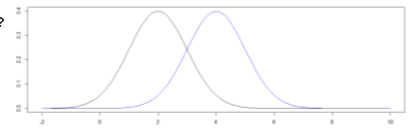
Prediction

Suppose we are to observe a random variable $X \sim N(\theta\,,1)$

And θ is known to be either 2 or 4.

What is the pdf of X?



Weight of male and female kestrels in 00s grams

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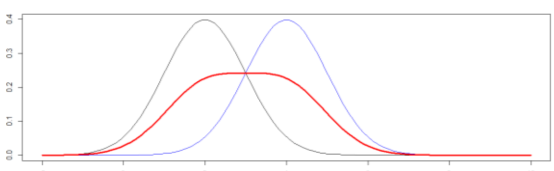
If both options are equally likely, then let's just average



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General rules for prediction

We condition on the value for the unknown parameter then multiply by the probability

$$p(\theta_1) \times p(x|\theta = \theta_1) + p(\theta_2) \times p(x|\theta = \theta_2)$$

Supposing θ has its own pdf…

Then,

$$p(x) = \int p(x|\theta)p(\theta)\,\mathrm{d}\theta$$

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Intro to Practical Session (Part I)

- On a summers' day, I am counting the number of large white butterflies in my garden
- I've been sat observing for a short while, but I am hungry and want some lunch.
- Before I slip away for lunch, I need to know how likely it is that I will miss an observation of a large white butterfly.

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Practical Session

See the practical session link for session 2a

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The main problem

To answer the question, the primary practical problem is to calculate $P(\theta|y)$

Analytical solutions are relatively rare (exceptions include use of conjugate priors

So… there are 2 general approaches:

1. Use an analytical approximate solution/method
2. Use simulation (Monte Carlo methods)

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Approximation methods

Beyond the scope of this course. Generally referred to as Approximate Bayesian Computation (ABC)

Specific variants of ABC include:

Laplace approximation (this is generally based on normal approximations)

and

Variational Inference

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Sampling

So, we are interested in some unknown parameters and have observed some data.

Bayes' theorem lets us switch this around and think about the parameters in terms of the data conditional on the parameters themselves

Why is this helpful?

Because what do we normally do in statistics when there is an unknown to quantify?

We **sample**!!

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The basics

- Sample a value of the parameter and calculate the prior distribution $p(\theta)$
- Conditional on this, calculate the likelihood $p(x|\theta)$
- Evaluate the posterior $p(\theta|x) = p(\theta) \times p(x|\theta)$
- Retain or reject the value of θ based on the posterior

David will go through this in more detail after the break

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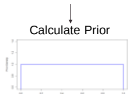
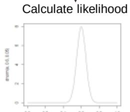
A schematic for sampling

Choose candidate

Calculate Prior

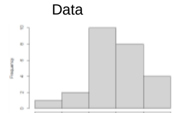


Calculate likelihood



Calculate posterior

Data



Retain parameter value or discard

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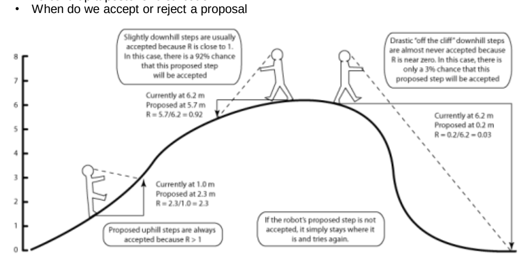
Intro to Practical Session (Part II)

Image © MCMC robot

Sampling
- To build up a posterior distribution
- When do we accept or reject a proposal



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Practical Session

See the practical session link for session 2a

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Key learning points

What are your key learning points from this session?

Can we identify similarities and differences with frequentist approach?

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Any Questions?

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This concludes…

Some more Bayesian basics

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Time for a break

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